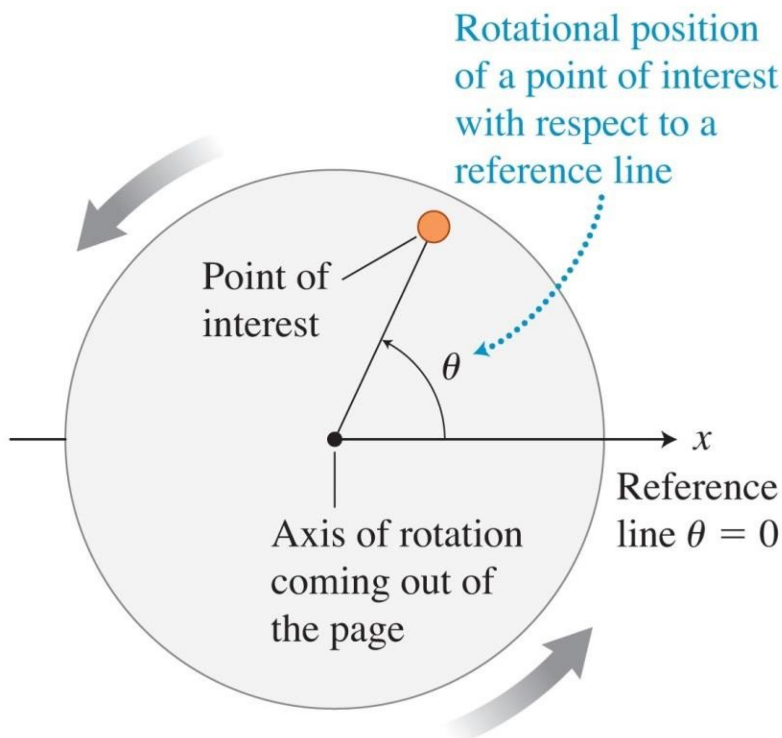
AP Physics 1 Exam Review
Session 3

Rotational Mechanics

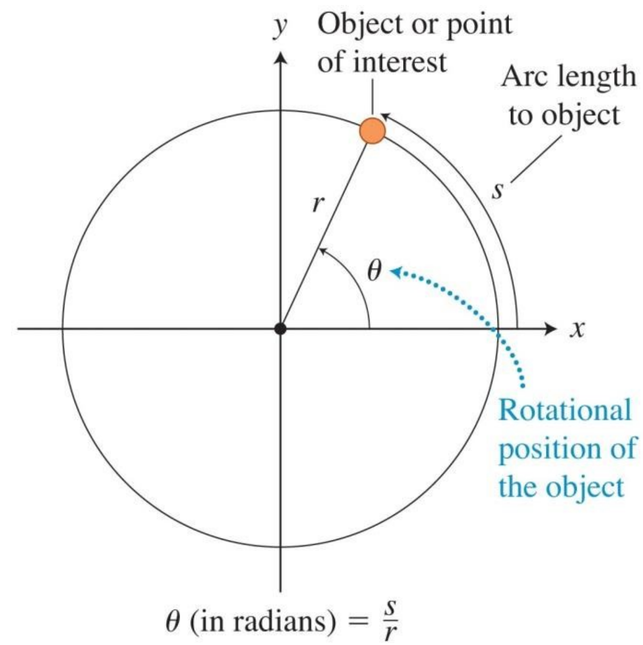
Rotational (angular) position ϑ



Rotational position θ The rotational position θ of a point on a rotating object (sometimes called the angular position) is defined as an angle in the counterclockwise direction between a reference line (usually the positive x-axis) and a line drawn from the axis of rotation to that point. The units of rotational position can be either degrees or radians.

Units of rotational position

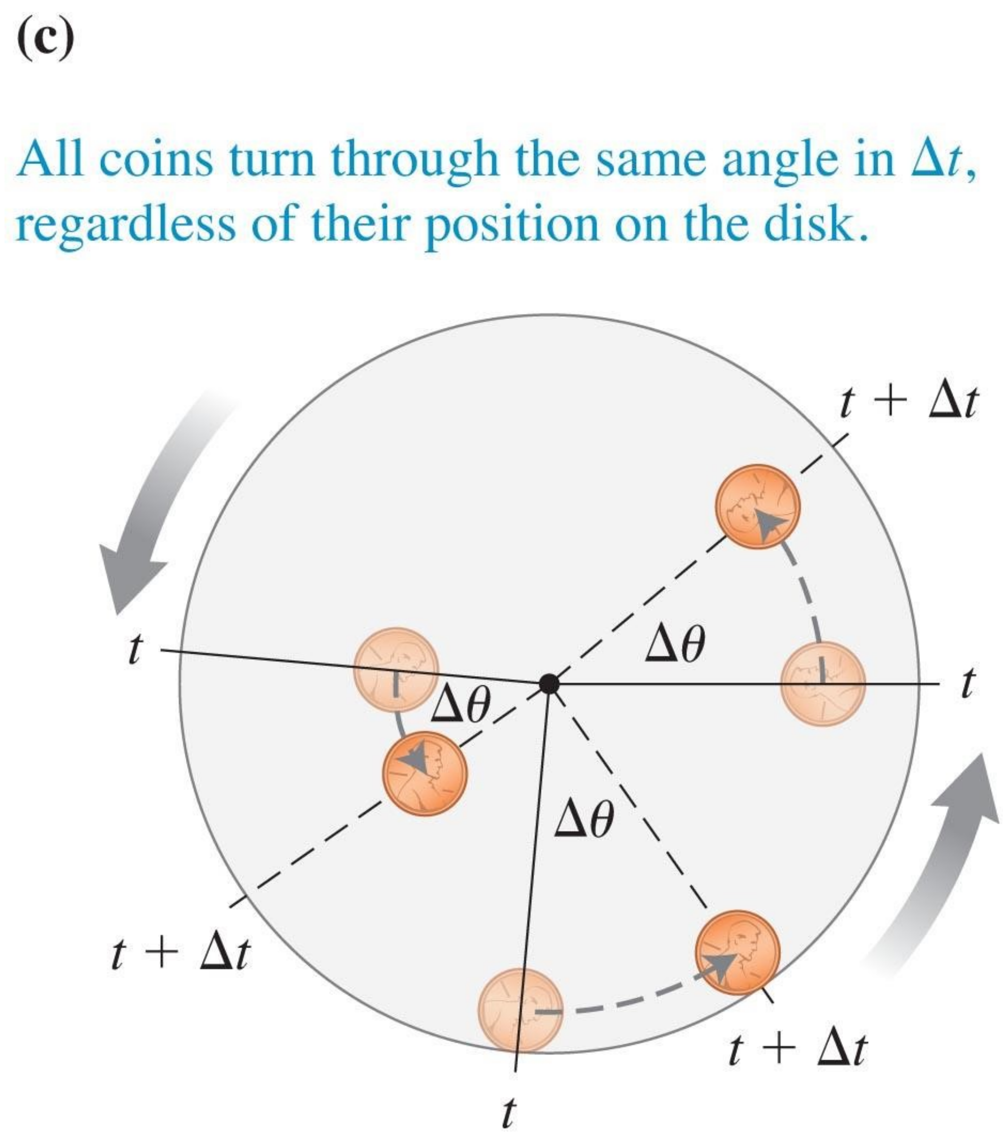
- The unit for rotational position is the radian (rad). It is defined in terms of:
  - The arc length *s*
  - The radius *r* of the circle



- The angle in units of radians is the ratio of *s* and *r*:
- The radian unit has no dimensions; it is the ratio of two lengths. The unit **rad** is just a reminder that we are using radians for angles.

Rotational (angular) velocity ω

- Translational velocity is the rate of change of linear position.
- We define the rotational (angular) velocity v of a rigid body as the rate of change of each point's rotational position.
  - All points on the rigid body rotate through the same angle in the same time, so **each point has the same rotational velocity**.



Rotational (angular) velocity ω

Rotational velocity ω The average rotational velocity (sometimes called angular velocity) of a turning rigid body is the ratio of its change in rotational position $\Delta\theta$ and the time interval Δt needed for that change (see **Figure 8.5**):

$$\omega = \frac{\Delta\theta}{\Delta t} \tag{8.2}$$

The sign of ω (omega) is positive for counterclockwise turning and negative for clockwise turning, as seen looking along the axis of rotation. *Rotational (angular) speed* is the magnitude of the rotational velocity. The most common units for rotational velocity and speed are radians per second (rad/s) and revolutions per minute (rpm).

Rotational (angular) acceleration α

- Translational acceleration describes an object's change in velocity for linear motion.
  - We could apply the same idea to the center of mass of a rigid body that is moving as a whole from one position to another.
- The rate of change of the rigid body's rotational velocity is its rotational acceleration.
  - When the rotation rate of a rigid body increases or decreases, it has a nonzero rotational acceleration.

Rotational (angular) acceleration α

Rotational acceleration α The average rotational acceleration α (alpha) of a rotating rigid body (sometimes called angular acceleration) is its change in rotational velocity $\Delta\omega$ during a time interval Δt divided by that time interval:

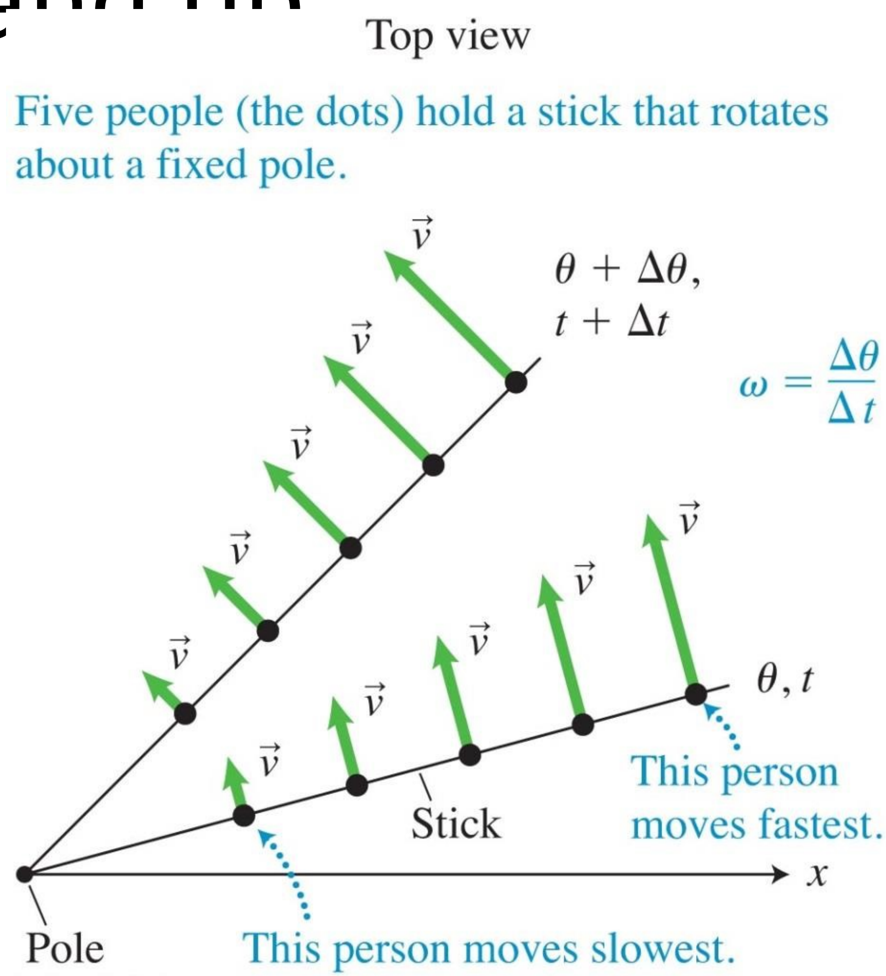
$$\alpha = \frac{\Delta\omega}{\Delta t} \tag{8.3}$$

The unit of rotational acceleration is $(\text{rad/s})/\text{s} = \text{rad/s}^2$.

Relating translational and rotational quantities and tip

$$v_t = \frac{\Delta s}{\Delta t} = \frac{r\,\Delta\theta}{\Delta t} = r\left(\frac{\Delta\theta}{\Delta t}\right) = r\omega$$

$$a_t = \frac{\Delta v_t}{\Delta t} = \frac{r\,\Delta\omega}{\Delta t} = r\left(\frac{\Delta\omega}{\Delta t}\right) = r\alpha$$



TIP You get the familiar translational quantities for motion along the circular path by multiplying the corresponding angular rotational quantities by the radius *r* of the circle.

Rotational motion at constant acceleration

- ϑ_0 is an object's rotational position at $t_0 = 0$.
- ω_0 is an object's rotational velocity at $t_0 = 0$.
- ϑ and ω are the rotational position and the rotational velocity at some later time t.
- α is the object's constant rotational acceleration during the t

Table 8.1 **Equations of kinematics for translational motion with constant acceleration and the analogous equations for rotational motion with constant rotational acceleration.**

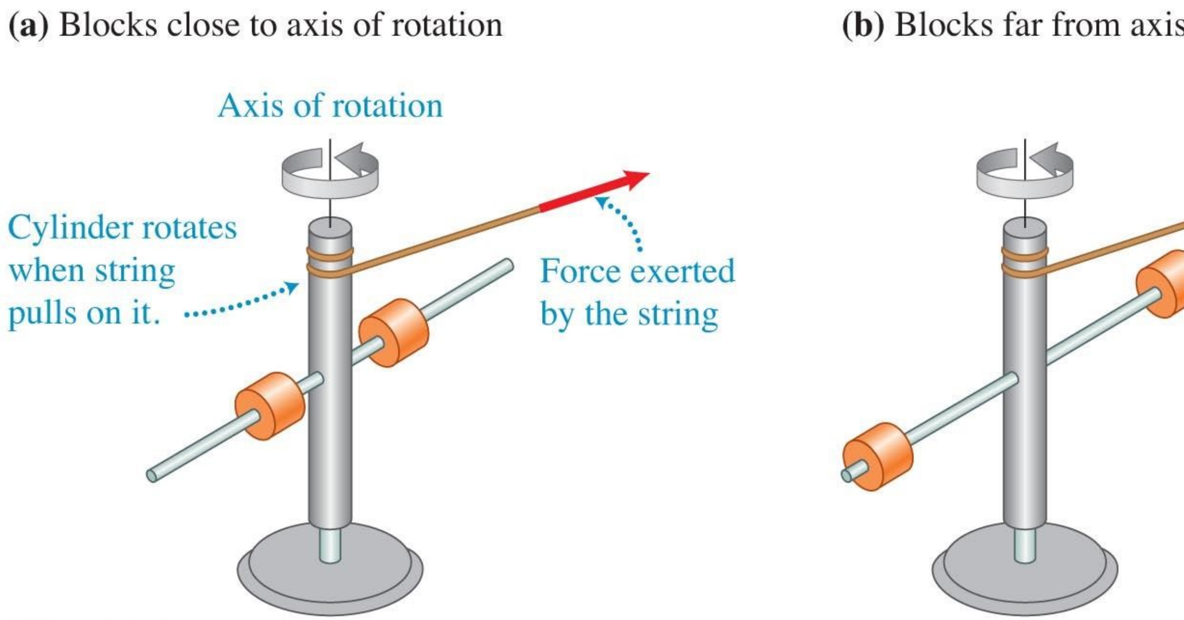
Translational motion	Rotational motion	
$v_x = v_{0x} + a_x t$	$\omega = \omega_0 + \alpha t$	(8.6)
$x = x_0 + v_{0x} t + \frac{1}{2} a_x t^2$	$\theta = \theta_0 + \omega_0 t + \frac{1}{2} \alpha t^2$	(8.7)
$2a_x(x - x_0) = v_x^2 - v_{0x}^2$	$2\alpha(\theta - \theta_0) = \omega^2 - \omega_0^2$	(8.8)

Equations Provided

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$$

$$\omega = \omega_0 + \alpha t$$

Rotational inertia



- Pull each string, and compare the rotational acceleration for the arrangement shown on the left and the right:
  - Our pattern predicts that the rotational acceleration will be greater for the arrangement on the left because the mass is nearer to the axis of rotation.
  - When we try the experiment, we find this to be true, consistent with our pattern.

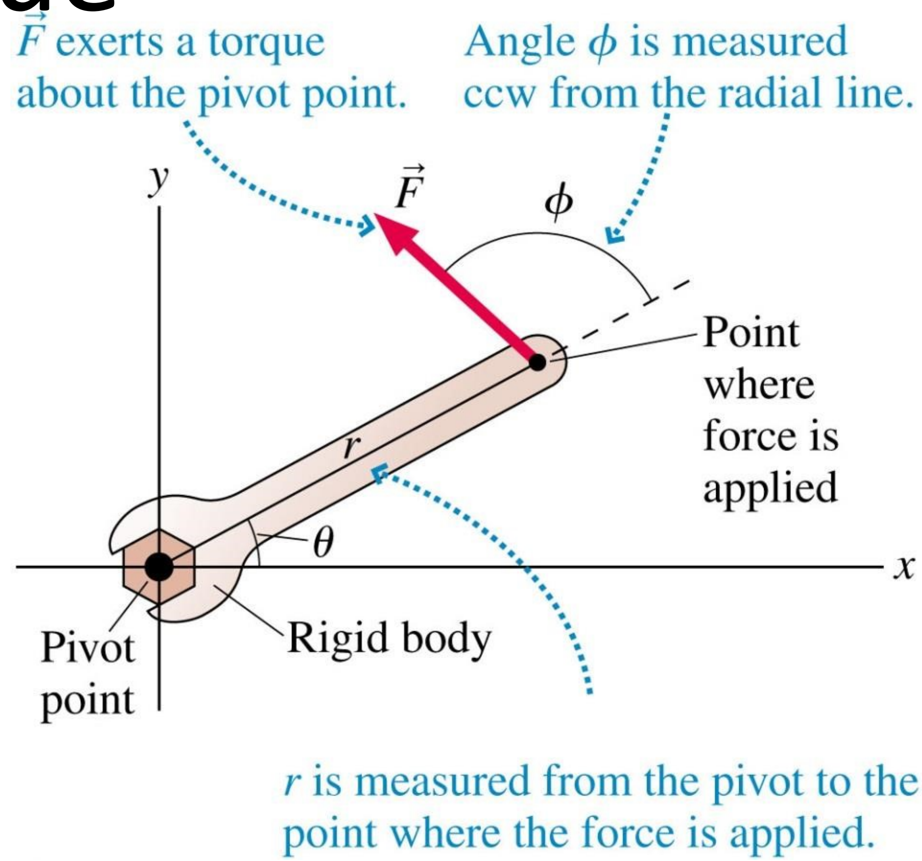
Rotational inertia

- Rotational inertia is the physical quantity characterizing the location of the mass relative to the axis of rotation of the object.
  - The closer the mass of the object is to the axis of rotation, the easier it is to change its rotational motion and the smaller its rotational inertia.
  - The magnitude depends on both the total mass of the object and the distribution of that mass about its axis of rotation.

Torque

Mathematically, we define torque τ (Greek tau) as

$$\tau \equiv rF\sin\phi$$



SI units of torque are N m.
English units are foot-pounds.

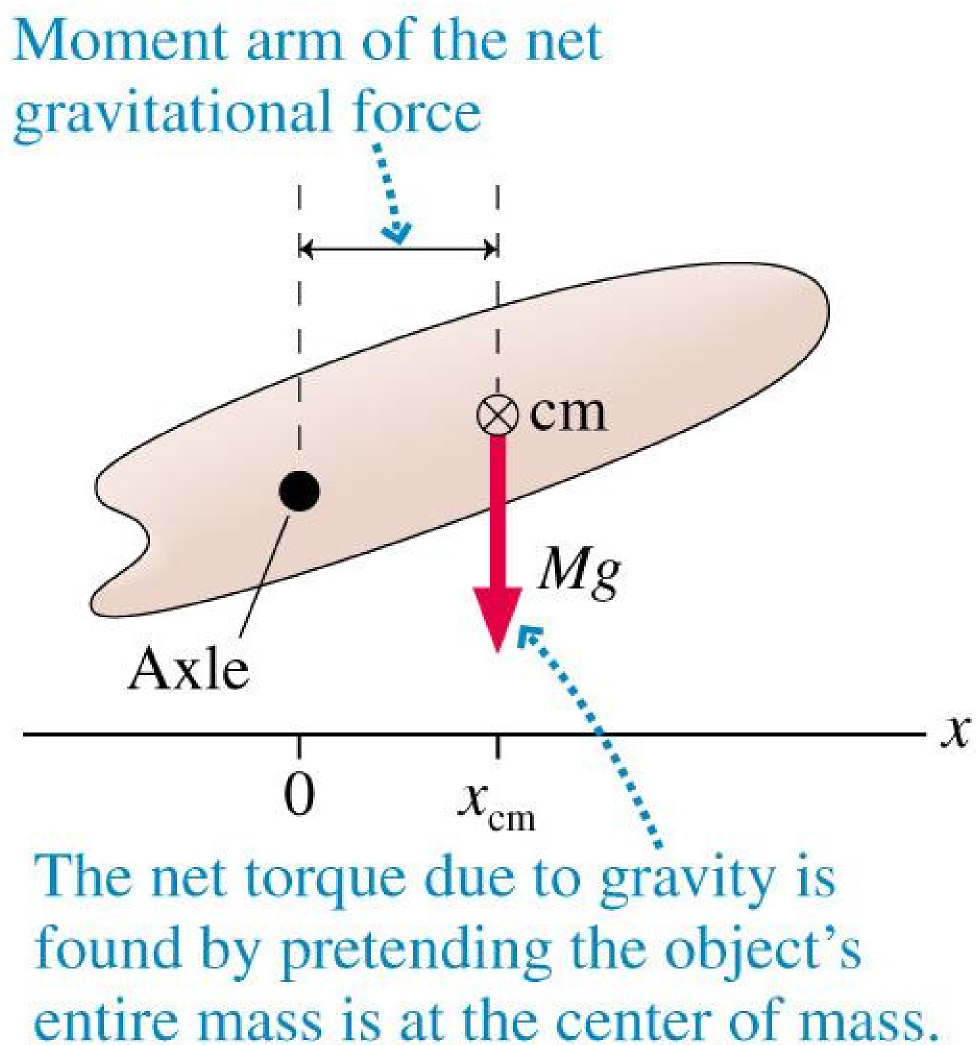
The ability of a force to cause a rotation depends on

1. the magnitude F of the force.
2. the distance r from the point of application to the pivot.
3. the angle at which the force is applied.

Gravitational Torque

The torque due to gravity is found by treating the object as if all its mass is concentrated at the center of mass.

$$\tau_{\text{grav}} = -Mgx_{\text{cm}}$$



Rotational Dynamics

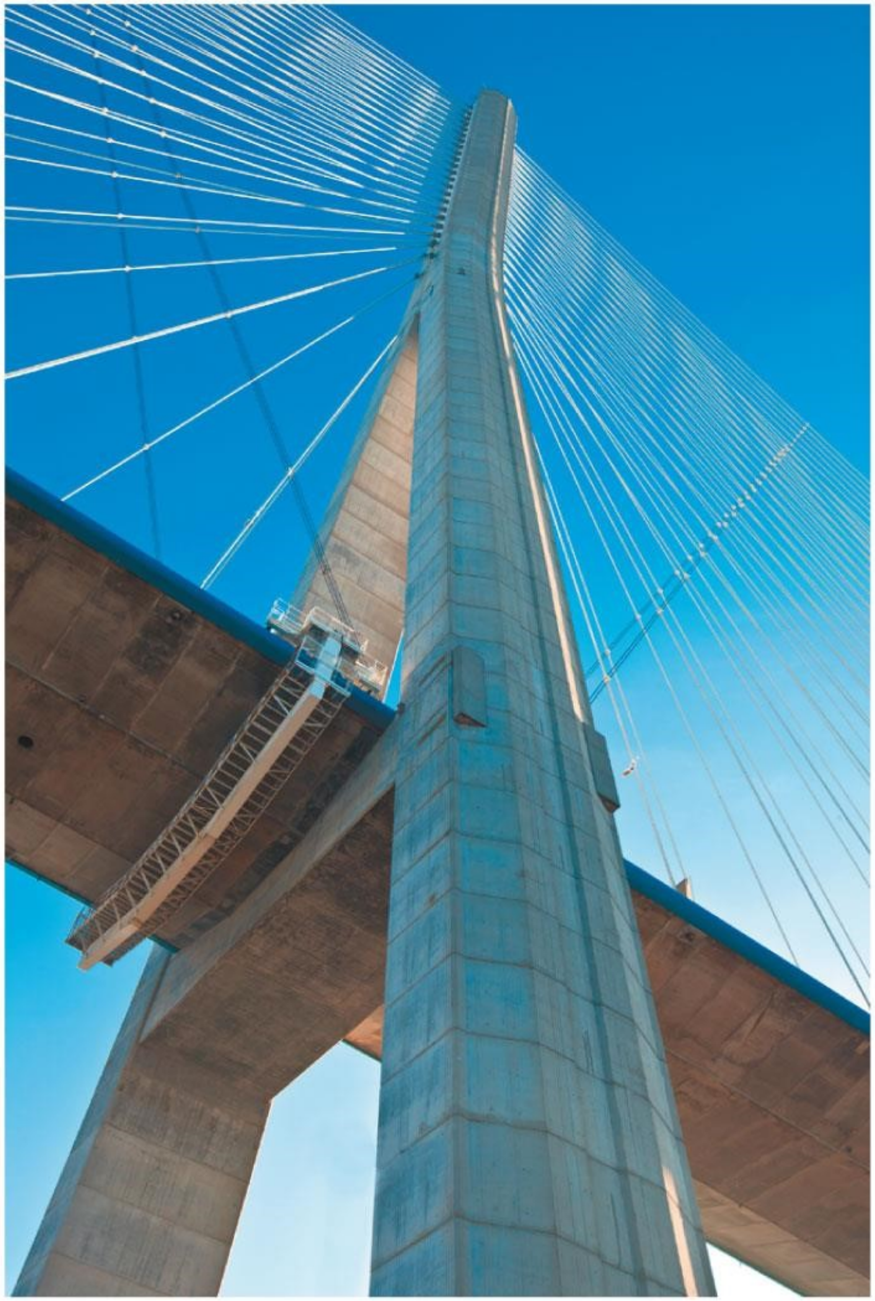
- What does a torque do?
- For linear motion, a net force causes an object to accelerate.
- For rotation, **a net torque causes an object to have angular acceleration.**

$$\alpha = \frac{\tau_{net}}{I} \qquad \text{(Newton's second law for rotational motion)}$$

In the absence of a net torque ($\tau_{net} = 0$), the object either does not rotate ($\omega = 0$) or rotates with *constant* angular velocity ($\omega = \text{constant}$).

Static Equilibrium

- A rigid body is in *static equilibrium* if there is **no net force** and **no net torque.**
- An important branch of engineering called *statics* analyzes buildings, dams, bridges, and other structures in total static equilibrium.
- **For a rigid body in total equilibrium, there is no net torque about any point.**



Equations Provided

$$\vec{\alpha} = \frac{\sum \vec{\tau}}{I} = \frac{\vec{\tau}_{net}}{I}$$

$$\tau = r_{\perp} F = rF \sin\theta$$

Rotational kinetic energy

Rotational kinetic energy The rotational kinetic energy of an object with rotational inertia I turning with rotational speed $\boldsymbol{\omega}$ is

$$K_{\text{rotational}} = \frac{1}{2}I\omega^2 \qquad (8.14)$$

TIP When you encounter a new physical quantity, always check whether its units make sense. In this particular case the units for I are $\text{kg} \cdot \text{m}^2$ and the units for ω^2 are $1/\text{s}^2$. Thus, the unit for kinetic energy is $\text{kg} \cdot \text{m}^2/\text{s}^2 = (\text{kg} \cdot \text{m}/\text{s}^2)\text{m} = \text{N} \cdot \text{m} = \text{J}$, the correct unit for energy.

Kinetic Energy of Rolling

The kinetic energy of a rolling object is

$$K_{\text{rolling}} = \frac{1}{2} I_{\text{cm}} \omega^2 + \frac{1}{2} M v_{\text{cm}}{}^2 = K_{\text{rot}} + K_{\text{cm}}$$

In other words, the rolling motion of a rigid body can be described as a translation of the center of mass (with kinetic energy K_{cm}) plus a rotation about the center of mass (with kinetic energy K_{rot}).

Equations Provided

$$K = \frac{1}{2} I\omega^2$$

Angular Momentum

- Unit is kg m^2/s
- $L = I\,\omega$
- Changes when a net torque is applied over time

- Always conserved in a closed system

$$L_0 = L_f$$

Conservation of Angular Momentum

- As an ice skater spins, external torque is small, so her angular momentum is almost constant.
- By drawing in her arms, the skater reduces her moment of inertia I.
- To conserve angular momentum, her angular speed ω must increase .

Equations Provided

$$L = I\omega$$

$$\Delta L = \tau \, \Delta t$$