

Angular Kinematics Practice

$$\omega = \frac{\Delta\theta}{t} \quad \alpha = \frac{\Delta\omega}{t} \quad \Delta\theta = \frac{\Delta x}{r} \quad \omega = \frac{v}{r} \quad \omega = \frac{2\pi}{T} \quad \alpha = \frac{a_T}{r} \quad a_c = \frac{v^2}{r} = \omega^2 r$$

$$v = \frac{2\pi r}{T} \quad \omega_f = \omega_0 + \alpha t \quad \Delta\theta = \frac{1}{2}(\omega_0 + \omega_f)t \quad \Delta\theta = \omega_0 t + \frac{1}{2}\alpha t^2 \quad \omega_f^2 = \omega_0^2 + 2\alpha\Delta\theta \quad T = \frac{1}{f}$$

- Determine the linear distance traveled for an object that makes 2.5 revolutions in a circular path with a radius of curvature of 0.8 m. **12.56 m**
- If, from question #1, this distance is covered in 0.55 s, what is the average angular velocity in radians per second? **28.56 rad/s**
- An object has a centripetal acceleration of 5.6 m/s². Determine the angular velocity of this object if the radius of the circle is 0.75 m. **2.73 rad/s**
- From the previous question, determine the period of revolution for the object. **2.3 s**
- If an object's tangential acceleration is 1.2 m/s², what is its angular displacement after 4.5 s if it starts from rest and travels along a path where the radius is 0.66 m? **18.4 rad**
- What is an object's angular velocity after 3.4 s while accelerating at 1.1 rad/s² if it starts from rest? **3.74 rad/s**
- From the previous question, what is its angular displacement during this acceleration? **6.36 rad**
- A ball is swung horizontally with an increasing velocity. If it begins from rest, and completes 4.3 revolutions per second after making 24 revolutions, what is the angular acceleration of the object in both rev/s² and rad/s²? **0.39 rev/s²; 2.42 rad/s²**
- A bicycle has 0.750 m diameter wheels. The bicyclist accelerates from rest with constant acceleration to 6.67 m/s in 14.0 s. What is the angular acceleration of the wheels? **1.27 rad/s²**
- To start a lawn mower, you must pull on a rope wound around the perimeter of a flywheel. After you pull the rope for 0.95 s, the flywheel is rotating at 4.5 rev/s, at which the rope disengages. This attempt at starting the mower does not work, and the flywheel slows to rest in 0.24 s. Assume constant accelerations occur.
 - Determine the average angular acceleration during the spin up, and the average angular acceleration during the spin down. **29.76 rad/s²; -117.9 rad/s²**
 - What is the maximum angular speed in rad/s reached by the flywheel? **28.27 rad/s**
- When a turntable rotating at 33 rpm is shut off, it comes to rest in 26 s. Assuming constant angular acceleration, find (a) the angular acceleration, (b) the average angular speed, and (c) the angular displacement in radians. **-0.133 rad/s²; 1.73 rad/s; 44.92 rad**
- A Ferris wheel with a radius of 12 m rotates once every 27 s, moving at a constant speed. What is its angular speed in radians per second? What is the linear speed of a passenger? What is the acceleration of a passenger (Think about which kind of acceleration she is experiencing.)? **0.23 rad/s; 2.79 m/s; 0.65 m/s²**

ANGULAR KINEMATICS PRACTICE SOLUTIONS

① $\Delta\theta = 2.5 \text{ rev} \times \frac{2\pi \text{ rad}}{1 \text{ rev}} = 5\pi \text{ rad}$ $\Delta\theta = \frac{\Delta x}{r}$
 $r = 0.8 \text{ m}$ $5\pi \text{ rad} = \frac{\Delta x}{(0.8 \text{ m})}$
 $\Delta x = ?$ $\Delta x = 12.56 \text{ m}$

② $\Delta\theta = 5\pi \text{ rad}$ $\omega = \frac{\Delta\theta}{t}$
 $t = 0.55 \text{ s}$ $\omega = \frac{5\pi \text{ rad}}{0.55 \text{ s}}$
 $\omega = ?$ $\omega = 28.56 \text{ rad/s}$

③ $a_c = 5.6 \text{ m/s}^2$ $a_c = \frac{v^2}{r}$ $\omega = \frac{v}{r}$
 $r = 0.75 \text{ m}$ $5.6 \text{ m/s}^2 = \frac{v^2}{0.75 \text{ m}}$ $\omega = \frac{2.05 \text{ m/s}}{0.75 \text{ m}}$
 $\omega = ?$ $v = 2.05 \text{ m/s}$ $\omega = 2.73 \text{ rad/s}$

④ $\omega = 2.73 \text{ rad/s}$ $\omega = \frac{2\pi}{T}$
 $2.73 \text{ rad/s} = \frac{2\pi \text{ rad}}{T}$
 $T = 2.30 \text{ s}$

⑤ $a_T = 1.2 \text{ m/s}^2$ $\Delta x = v_0 t + \frac{1}{2} a t^2$ $\Delta\theta = \frac{\Delta x}{r}$
 $t = 4.5 \text{ s}$ $\Delta x = \frac{1}{2} (1.2 \text{ m/s}^2) (4.5 \text{ s})^2$ $\Delta\theta = \frac{12.15 \text{ m}}{0.66 \text{ m}}$
 $v_0 = 0$ $\Delta x = 12.15 \text{ m}$ $\Delta\theta = 18.4 \text{ rad}$
 $r = 0.66 \text{ m}$
 $\Delta\theta = ?$

$$\begin{aligned} \textcircled{6} \quad \omega_0 &= 0 \\ \alpha &= 1.1 \text{ rad/s}^2 \\ t &= 3.4 \text{ s} \\ \omega_f &= ? \end{aligned}$$

$$\begin{aligned} \omega_f &= \omega_0 + \alpha t \\ \omega_f &= (1.1 \text{ rad/s}^2)(3.4 \text{ s}) \\ \omega_f &= 3.74 \text{ rad/s} \end{aligned}$$

$$\textcircled{7} \quad \Delta\theta = ?$$

$$\begin{aligned} \Delta\theta &= \cancel{\omega_0 t} + \frac{1}{2} \alpha t^2 \\ \Delta\theta &= \frac{1}{2} (1.1 \text{ rad/s}^2) (3.4 \text{ s})^2 \\ \Delta\theta &= 6.36 \text{ rad} \end{aligned}$$

$$\begin{aligned} \textcircled{8} \quad \omega_0 &= 0 \\ \omega_f &= 4.3 \frac{\text{rev}}{\text{s}} \\ \Delta\theta &= 24 \text{ rev} \\ \alpha &= ? \end{aligned}$$

$$\begin{aligned} \omega_f^2 &= \cancel{\omega_0^2} + 2\alpha\Delta\theta \\ (4.3 \text{ rev/s})^2 &= 2\alpha(24 \text{ rev}) \\ \alpha &= 0.39 \text{ rev/s}^2 \left(\frac{2\pi \text{ rad}}{1 \text{ rev}} \right) = 2.42 \text{ rad/s}^2 \end{aligned}$$

$$\begin{aligned} \textcircled{9} \quad r &= 0.375 \text{ m} \\ v_i &= 0 \\ v_f &= 6.67 \text{ m/s} \\ t &= 14 \text{ s} \\ \alpha &= ? \end{aligned}$$

$$\begin{aligned} v_f &= v_0 + at \\ 6.67 \text{ m/s} &= a(14 \text{ s}) \\ a &= 0.48 \text{ m/s}^2 \end{aligned}$$

$$\begin{aligned} \alpha &= \frac{a}{r} \\ \alpha &= \frac{0.48 \text{ m/s}^2}{0.375 \text{ m}} \\ \alpha &= 1.28 \text{ rad/s}^2 \end{aligned}$$

(10) a) Spin up
 $\omega_0 = 0$

$$\omega_f = 4.5 \text{ rev/s}$$

$$t = 0.95 \text{ s}$$

$$\alpha = ?$$

$$\omega_f = \omega_0 + \alpha t$$

$$4.5 \text{ rev/s} = \alpha (0.95 \text{ s})$$

$$\alpha = 4.74 \text{ rev/s}^2 = \left(\frac{2\pi \text{ rad}}{1 \text{ rev}} \right) 29.76 \text{ rad/s}^2$$

Spin down

$$\omega_0 = 4.5 \text{ rev/s}$$

$$\omega_f = 0$$

$$t = 0.24 \text{ s}$$

$$\alpha = ?$$

$$\omega_f = \omega_0 + \alpha t$$

$$0 = (4.5 \text{ rev/s}) + \alpha (0.24 \text{ s})$$

$$\alpha = -18.8 \text{ rev/s}^2 = \left(\frac{2\pi \text{ rad}}{1 \text{ rev}} \right) -117.8 \text{ rad/s}^2$$

b) $\omega = 4.5 \text{ rev/s} \left(\frac{2\pi \text{ rad}}{1 \text{ rev}} \right) = 28.27 \text{ rad/s}$

(11) $\omega_0 = \frac{33 \text{ rev}}{\text{min}} \left(\frac{2\pi \text{ rad}}{1 \text{ rev}} \right) \left(\frac{1 \text{ min}}{60 \text{ s}} \right) = 3.46 \text{ rad/s}$

$$\omega_f = 0$$

$$t = 26 \text{ s}$$

a) $\alpha = ?$

$$\omega_f = \omega_0 + \alpha t$$

$$0 = 3.46 \text{ rad/s} + \alpha (26 \text{ s})$$

$$\alpha = -0.13 \text{ rad/s}^2$$

b) $\omega_{\text{avg}} = \frac{1}{2} (\omega_0 + \omega_f)$

$$\omega_{\text{avg}} = \frac{1}{2} (3.46 \text{ rad/s} + 0)$$

$$\omega_{\text{avg}} = 1.73 \text{ rad/s}$$

c) $\Delta\theta = ?$

$$\Delta\theta = \omega_0 t + \frac{1}{2} \alpha t^2$$

$$\Delta\theta = (3.46 \text{ rad/s})(26 \text{ s}) + \frac{1}{2} (-0.13 \text{ rad/s}^2)(26 \text{ s})^2$$

$$\Delta\theta = 46.02 \text{ rad}$$

$$(12) \quad r = 12 \text{ m}$$

$$T = 27 \text{ s}$$

$$\omega = ?$$

$$v = ?$$

$$a_c = ?$$

$$\omega = \frac{2\pi}{T}$$

$$\omega = \frac{2\pi \text{ rad}}{27 \text{ s}}$$

$$\omega = 0.233 \text{ rad/s}$$

$$v = \omega r$$

$$v = (0.233 \text{ rad/s})(12 \text{ m})$$

$$v = 2.80 \text{ m/s}$$

$$a_c = \frac{v^2}{r} = \frac{(2.80 \text{ m/s})^2}{12 \text{ m}} = 0.65 \text{ m/s}^2$$